

On the question of trapped surfaces and black holes

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Abstract

There are many observational evidences for the detection of compact objects with masses significantly larger (in galactic cases) or much larger (in extra-galactic cases) than the upper limits of masses of *cold* Neutron Stars. Such compact objects are commonly interpreted as Black Holes (BHs). However, we point out that while such Black Hole Candidates (BHCs) must be similar to BHs in many respects they, actually, can not be BHs because existence of Black Holes would violate the basic tenet of the General Theory of Relativity that the *worldline of a material particle must be timelike at any regular region of spacetime*. We arrive at this unique conclusion by approaching the problem from various directions. We feel that such “operational” Black Holes could be able to explain hard X-ray tail found in the galactic BHCs because Lorentz factor of the plasma accreting on such objects should be considerably higher than the corresponding NS case.¹

I. INTRODUCTION

This is a Workshop on “Black Holes” and there are many good reasons why the present astrophysical community, in general, believes in the existence of BHs. In several X-ray binaries there are evidences for the existence of compact stars with masses larger than $4 - 5M_{\odot}$, the broad upper limit of masses of cold baryonic objects in the standard Quantum Chromo Dynamics (QCD). Similarly there are evidences that the core of many normal galaxies (like the Milky Way) and Active galaxies contain massive dark condensations with masses ranging from $10^6 - 10^{10}M_{\odot}$. Currently the best explanation for such condensations is that they are supermassive BHs. There are other physical reasons for assuming the existence of BHs which will be mentioned towards the end of this article. However, we shall see that the General Theory of Relativity (GTR) actually does not allow the existence of BHs and thus while these BHCs must be operationally similar to BHs, in a strict sense, they must be different from BHs. And we plead that the reader keeps an open mind about this whole discussion.

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II. SPHERICALLY SYMMETRIC GRAVITATIONAL FIELD

Any spherically symmetric metric, upon, suitable coordinate transformation, can be brought to a form [1]:

$$ds^2 = g_{00}dx_0^2 + g_{11}dx_1^2 - R^2(x_1, x_0)(d\theta^2 + \sin^2 \theta d\phi^2) \quad (1)$$

Here x_1 is the general radial coordinate (normally, $g_{11} < 0$ and x_0 is the general temporal coordinate (normally, $g_{00} > 0$ [1]) because the chosen signature of the metric is (+1, -1, -1, -1). Here R is the *Invariant Circumference Coordinate* (a scalar). R being a scalar, it must retain its essential property as a “Spacelike” coordinate. Since, we shall deal with only radial worldlines with $d\theta = d\phi = 0$, our effective metric will be

$$ds^2 = g_{00}dx_0^2 + g_{11}dx_1^2 \quad (2)$$

Following Landau & Lifshitz [1], we can rewrite it as

$$ds^2 = g_{00}dx_0^2[1 - V^2] \quad (3)$$

where $V \equiv \frac{\sqrt{-g_{11}}dx_1}{\sqrt{g_{00}}dx_0}$. Here we take speed of light $c = 1$. Although the physical meaning of V obvious, in order to highlight that our conclusions do not depend on this physical meaning, we will not even mention it. The element of proper time measured by a $x_1 = \text{constant}$ observer is $d\tau = \sqrt{g_{00}}dx_1$ and the element of proper radial distance is $dl = \sqrt{-g_{11}}dx_0$. Therefore, we may rewrite

$$V = \frac{dl}{d\tau} \quad (4)$$

Since the worldline of a material particle must be non-spacelike everywhere (even at a true singularity) $ds^2 \geq 0$, from Eq.(3), it follows that, we must have $g_{00}(1 - V^2) \geq 0$. Thus if $V \leq 1$, we must have $g_{00} \geq 0$, and if $V \geq 1$, we must have $g_{00} \leq 0$. Also, the determinant of the metric coefficients $g = R^4g_{11}g_{00}\sin^2 \theta$ is always negative [1] or zero at a singularity; or $g \leq 0$, in general. Thus, if $V \leq 1$, $g_{00} \geq 0$ and $g_{11} \leq 0$. On the other hand, if $V \geq 1$, $g_{00} \leq 0$ and $g_{11} \geq 0$. Thus *under any circumstances*, we must have $-(1 - V^2)/g_{11} \geq 0$.

III. SPHERICAL GRAVITATIONAL COLLAPSE

If BHs are there, they must have resulted from gravitational collapse of stars or other fluids. The spherical collapse is studied best in the socalled comoving coordinates, r, t , where $x_1 = r$ is label attached to a fluid shell with a fixed number of baryons and $x_0 = t$. It follows from the general formalism of gravitational collapse [2–8] that the integration of the 0,0 component of the Einstein equation leads to a constraint

$$\Gamma^2 = 1 + U^2 - 2GM(r)/R \quad (5)$$

where G is the gravitational constant and $M(r)$ is the gravitational mass enclosed by a shell with $r = r$. Here the parameters

$$\Gamma \equiv \frac{1}{\sqrt{-g_{rr}}} \frac{\partial R}{\partial r}; \quad U \equiv \frac{1}{\sqrt{g_{00}}} \frac{\partial R}{\partial t} \quad (6)$$

Note that while Γ is a partial derivative w.r.t. r , it is a total derivative w.r.t. l because the notion of a fixed t is steeped into the definition of proper radial length l . Also, while U is a partial derivative w.r.t. t , it is a total derivative w.r.t. τ because the notion of a fixed r is steeped into the definition of proper time [2–8]:

$$\Gamma = \frac{dR}{dl}; \quad U = \frac{dR}{d\tau} \quad (7)$$

To fully appreciate this subtle point, the reader is specifically referred to (see pp. 180-181 of ref. 6 and pp. 150-151 of ref. 7). From Eqs. (5) and (8) we note that U and Γ are interlinked as

$$U = \Gamma V \quad (8)$$

By inserting the above relation in Eq.(6), and by transposing, we find,

$$\Gamma^2(1 - V^2) = 1 - 2GM(r)/R \quad (9)$$

Or,

$$\frac{1}{-g_{rr}} \left(\frac{\partial R}{\partial r} \right)^2 (1 - V^2) = 1 - 2GM(r)/R \quad (10)$$

But in previous section we have already found that $-(1 - V^2)/g_{rr} \geq 0$. Therefore, the L.H.S. of the Eq.(11) is ≥ 0 . So must be its R.H.S. And hence, we must have

$$\frac{2GM(r)}{R} \leq 1; \quad \frac{R_g}{R} \leq 1 \quad (11)$$

On the other hand, the condition for formation of a “trapped surface” is that $2GM/R > 1$. Thus we find that, in spherical gravitational collapse *trapped surfaces do not form*. Note that this result *does not depend* on the physical interpretation of V or whether $V < 1$ or $V > 1$, or if there is a coordinate singularity or not. If the collapse process indeed continues upto $R = 0$, in order that the foregoing constraint is satisfied, we must have $M(r) \rightarrow 0$ as $R \rightarrow 0$. This means that the final singularity must be of zero *gravitational mass* if we assume positivity of mass. Remember here that the quantity M is not the fixed baryonic mass : $M \neq M_0 = mN$. Physically, the $M = 0$ state may result when the *negative gravitational energy* exactly cancels the internal energy, the *baryonic mass energy* M_0 and any other energy. Since trapped surface is not formed the system keeps on radiating and M keeps on decreasing till the lowest value ($M = 0$) is reached [8]. We have found that the final state corresponds to $2GM/R \rightarrow 1$ rather than $2GM/R < 1$ [9]. This means that the final $M = 0$ state is enclosed by a horizon and there is thus no chance of attaining a negative mass state. We have also found that as the final state is attained, the proper radial length [9] $l \sim \int_0^R \sqrt{-g_{rr}} dr \rightarrow \infty$ as $R \rightarrow 0$ and $-g_{rr} \rightarrow \infty$. The corresponding *proper time* to attain the $R = 0$ state is $\tau \geq l/c \rightarrow \infty$. This means that the timelike worldlines are *geodesically complete* and at any finite *proper time* (not just coordinate time) sufficiently massive stars would be found as an Eternally Collapsing Object (ECO) rather than a BH.

IV. HOMOGENEOUS DUST COLLAPSE

It is widely believed that by studying the problem of the collapse of the most idealized fluid, i.e, a “dust” with pressure $p \equiv 0$ and no density gradient, Oppenheimer and Snyder (OS) [10] explicitly showed that finite mass BHs can be generated. We have discussed in detail [8] that this perception is completely incorrect. For the sake of brevity, we would like to mention here about the Eq.(36) of OS paper which connects the proper time T of a distant observer with a parameter $y = \frac{R}{2GM}$ (at the boundary $r = r_b$) through the Eq.

$$T \sim \ln \frac{y^{1/2} + 1}{y^{1/2} - 1} + \text{ other terms.} \quad (12)$$

In order that T is definable, the argument of this logarithmic term must be non-negative, i.e, $y = \frac{R}{2GM} \geq 1$, or, $\frac{2GM}{R} \leq 1$, which is nothing but our Eq.(12). Thus even for the most idealized cases, trapped surfaces are not formed. Since OS assumed M to be finite even at $R = 0$, the final value of $y = 0$ in their case, and this may lead to non-sensical results. For example, the spatial metric coefficient e^λ of their paper should have been ∞ at the singularity. But, it is finite in their case. Unfortunately OS overlooked all such inconsistencies and ended their paper soon after Eq. (36).

However, one can still legitimately wonder, if one starts with a dust of finite mass M , *and if the dust does not radiate*, why the condition $2GM/R > 1$ would not be satisfied at appropriate time? The point is that if we assume $p = 0$, dust is really not a fluid, it is just a collection of incoherent finite number of N particles distributed symmetrically. If so, there are free spaces in between the dust particles and which is not the case for a “continuous” fluid. Therefore although the dust particles are symmetrically distributed around the centre of symmetry, in a strict sense, the distribution is not really isotropic. Then the assembly of *incoherent* dust particles may be considered as a collection of $N/2$ symmetric pair of particles. In GTR, a pair of particles accelerate each other and generate gravitational radiation unmindful of the presence of other *incoherent* pairs. Therefore, the gravitational mass of an accelerating dust is really not constant! In contrast a physical spherical fluid will behave like a coherent single body with zero quadrupole moment and will not emit any gravitational radiation.

V. FINITE MASS SCHWARZCHILD BH ?

Suppose some of the readers would just refuse to accept the above conclusion that gravitational collapse does not lead to the formation of (strict) finite mass BH. If so, let us assume, for the time being, the existence of a Schwarzschild BH of mass M and horizon size $R_g = 2M$ (now we set $G = 1$). The spacetime for $R \geq R_g$ is described by

$$ds^2 = dT^2(1 - 2M/R) - \frac{dR^2}{(1 - 2M/R)} - R^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad (13)$$

For radial geodesics of test particles, $d\theta = d\phi = 0$, and it follows that along the radial geodesic

$$\left(\frac{dR}{dT}\right)^2 = \frac{(1-2M/R)^2}{E^2} [E^2 - (1-2M/R)] \quad (14)$$

where E is the conserved energy per unit rest mass. As $R \rightarrow 2M$, foregoing Eq. shows that

$$\left(\frac{dR}{dT}\right)^2 \rightarrow (1-2M/R)^2 \quad (15)$$

By transposing this Eq., we find that

$$dT^2(1-2M/R) \rightarrow \frac{dR^2}{1-2M/R} = dz^2 \quad (\text{say}) \quad (16)$$

By inserting the above relation in Eq.(14), we find that, for a radial geodesic, as $R \rightarrow 2M$

$$ds^2 \rightarrow dz^2 - dz^2 \rightarrow 0 \quad (17)$$

This means that, the *timelike geodesic* would turn *null* if an EH would exist. But this is not possible if EH is really a regular region of spacetime. Thus, if we insist on having a BH, the only honourable solution here would be to recognize that the EH is actually a true singularity, i.e, the central singularity. This would mean that $R_g = 0$ or $M = 0$. So if we assume that there is a BH, the permissible value of its mass is $M = 0$ or else we may have finite mass objects which could be very similar to BHs in many ways, but which are not exactly BHs.

Here the reader must avoid one probable confusion: Though event horizon is a null hypersurface, here we are not concerned with the metric on this hypersurface. We are concerned here with the metric (ds^2) *along the geodesic of a test particle*. The fact that that $ds^2(EH) = 0$ on the hypersurface does not automatically mean that $ds^2 = 0$ along a geodesic intersecting the EH.

VI. KRUSKAL COORDINATES

Since ds^2 is an invariant and independent of coordinate system used in evaluating it, our above proof that if an EH would exist, the value of ds^2 evaluated along the geodesic of a test particle would tend to be zero, must be valid in all coordinate systems. Yet it would be worthwhile to obtain the same result using the Kruaskal coordinates which are believed to be describe both the exterior and interior of a Schwarzschild BH (SBH). For the exterior region, we have (Sectors I and III):

$$u = f_1(R) \cosh \frac{T}{4M}; \quad v = f_1(R) \sinh \frac{T}{4M}; \quad f_1(R) = \pm \left(\frac{R}{2M} - 1 \right)^{1/2} e^{R/4M} \quad R \geq 2M \quad (18)$$

And for the region interior to the horizon (Sector s II and IV), we have

$$u = f_2(R) \sinh \frac{T}{4M}; \quad v = f_2(R) \cosh \frac{T}{4M}; \quad f_2(R) = \pm \left(1 - \frac{R}{2M} \right)^{1/2} e^{R/4M}; \quad R \leq 2M \quad (19)$$

Here the +ve sign refers to “our universe” and the -ve sign refers to the “other universe” implied by Kruskal diagram. In either region we have

$$u^2 - v^2 = \left(\frac{R}{2M} - 1 \right) e^{R/2M} \quad \text{so that} \quad u^2 = v^2; \quad R = 2M \quad (20)$$

By differentiating Eq.(21) by T , we find

$$2u \frac{du}{dT} - 2v \frac{dv}{dT} = \frac{e^{R/2M}}{2M} \frac{R}{2M} \frac{dR}{dT} \quad (21)$$

From Eq.(16) we see that $dR/dT = 0$ at $R = 2M$, hence it follows from the above Eq.that

$$\frac{du}{dv} = \frac{v}{u}; \quad R = 2M \quad (22)$$

Now combining Eqs.(21) and (23), we find that

$$du^2 = dv^2; \quad R = 2M \quad (23)$$

Had we differentiated Eq.(21) by R or v instead of T we would have obtained the same result because it can be shown that

$$\frac{dv}{dR} = \frac{rv}{8M^2} (R/2M - 1)^{-1} + \frac{u}{4M} \frac{dT}{dR} = \infty; \quad R = 2M \quad (24)$$

In terms of u and v , the metric for the entire spacetime is

$$ds^2 = \frac{32M^3}{R} e^{-R/2M} (dv^2 - du^2) - R^2 (d\theta^2 + d\phi^2 \sin^2 \theta) \quad (25)$$

Then, using Eq.(24), we find that for a radial geodesic $ds^2 = 16M^2 e^{-1} (du^2 - du^2) = 0$ at $R = 2M$. This leads us to the same conclusion: there can not be any finite mass BH.

VII. COORDINATE SINGULARITY AT EH ?

One of the main arguments in favour of the idea that the EH is a mere coordinate singularity and not a physical singularity is that no physically meaningful scalars are singular there. The most cited example here is that of the curvature scalar $K = \frac{48M^2}{R^6}$. At $R = 2M$, it is reduced to $K = \frac{3}{4M^4}$. Thus K is finite only as long as M is finite. But we have found that, for a BH, $M \equiv 0$, and hence, as is expected, actually, K is singular at the EH. In the following, let us consider another physically meaningful scalar at the EH. By differentiating the 4-velocity, we can obtain the Acceleration 4-vector [1]

$$a^i \equiv \frac{Du^i}{Ds} = \frac{du^i}{ds} + \Gamma_{kl}^i u^k u^l \quad (26)$$

It follows that for a radial geodesic, this acceleration vector has only the radial component and it blows up at $R = 2M$ [12,13]. However, this result may be rejected by the proponents of BH hypothesis by insisting that a^R is coordinate dependent quantity and its blowing up

is simply another manifestation of “coordinate singularity” at the EH. But one can form a coordinate independent scalar by contracting a^i [12,13]: $a \equiv \sqrt{-a_i a^i}$. And it is found that

$$a = \frac{M}{R^2 \sqrt{1 - 2M/R}} \quad (27)$$

Thus this *scalar indeed blows up at EH* $R = 2M$. Further, a would become imaginary if there would be a spacetime beneath $R = 2M$. Again all these happenings show that the EH is the central singularity and there can not be any spacetime beneath it. Hence, mathematically, for a Schwarzschild BH, we must have $M = 0$.

VIII. SCHWARZSCHILD OR HILBERT SOLUTION

The original paper of Schwarzschild where he worked out the spacetime around a *point mass*, M has recently been translated into English by Loinger and Antoci [14]. Even before this Abrams [12] pointed out that, the original Schwarzschild solution is

$$ds^2 = dT^2 \left(1 - \frac{2M}{R}\right) - \frac{dR_*^2}{\left(1 - \frac{2M}{R}\right)} - R^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad (28)$$

where

$$R = [R_*^3 + (2M)^3]^{1/3} \quad (29)$$

The important point is that here R is not the radial variable, on the other hand the radial variable is R_* ; the *point mass is sitting at $R_* = 0$ and not at $R = 0$* . Thus at, $R_* = 2M$, obviously there is no singularity, whereas there is a central singularity at $R_* = 0$. Since there is no spacetime beneath $R_* < 0$, there is no spacetime beneath $R < 2M$. The prevalent Sch. solution, which is actually the Hilbert solution is, however, still a mathematically valid solution for a “point mass”. So the question, which is the physically valid solution, has to be decided on physical reasoning. Here, some authors [12,15] have rejected the Hilbert solution because according to it, there would be a spacetime beneath $R < 2M$ and the scalar a would then first blow up and then become imaginary. On this ground, these authors, claim that the original Schwarzschild solution is the physically valid solution. If so, in this picture, obviously, there would be no BH. On the other hand, there is a problem with the original Sch. solution; as admitted by Sch. himself [13], in the limit of weak gravity, his equation does not exactly reduce to a Newtonian form. On the other hand, we know that, the prevalent Sch solution, i.e, the Hilbert solution does yield the correct Newtonian form. Further, the angular part of either Schwarzschild or Hilbert solution (or any spherically symmetric metric) shows that R and not R_* is the Invariant Circumference Coordinate. Therefore, Hilbert solution can not really be rejected. Then how do we resolve this paradox? We note that, the problem lies with our premises that “there is a *point mass* with a finite gravitational mass”! And the solution lies in realizing that if there is a body of finite mass, it cannot be considered as a *geometrical point* even at a classical level.. On the other hand, if we insist that there is a point mass, its gravitational mass must be zero. When $M = 0$, Sch. solution and Hilbert soln. becomes identical, $R_* \equiv R$, and there is no EH,

but there is only a central singularity in a mathematical sense. One may note here that in classical electrodynamics there are “point charges”. And existence of “point charges” implies singularities. On the other hand, in quantum field theories, there are no point charges, for instance, an electron is actually not a “point charge”. We find here that in GTR, even at a classical, non-quantum level, there can not be any “point mass” (having finite M).

IX. SUMMARY AND CONCLUSIONS

We found that for continued collapse of any perfect fluid possessing arbitrary EOS and radiation transport properties, there is a global constraint which demands that (exact) trapped surfaces do not form. To appreciate this statement, let us recall the following: the depth of a gravitational potential well on a surface can be gauged by its gravitational redshift z . On a White Dwarf surface, $z \sim 10^{-4}$, on a NS surface, $z \sim 10^{-1}$. But on an EH, $z = \infty$. And our work shows that this $z = \infty$ stage is not realizable in any amount of finite *proper time* (not only in coordinate time). So at any finite proper time gravitational collapse produces compact objects with finite z where it is possible that $z \gg 0.1$ (the NS value). But such finite z objects need not always be static and cold, they need not represent stable solutions of equations for hydrostatic balance (Oppenheimer- Volkoff equation [16]). Luminosity of such an object would appear reduced by a factor of $(1+z)^2$ than its lower value (actually there would be reduction due to Doppler factor too). And thus for a sufficiently large value of z , such an Eternally Collapsing Object (ECO), may appear to approximately trap the light or radiation emitted by it. If gravitational collapse goes on and on, since, strictly, there is no trapped surface, the body would continue to emit radiation and lose (gravitational) mass. So eventually, asymptotically, after *infinite proper time*, the body would be reduced to a $M = 0$ state, and an EH would form only at this stage.

We found that the result that trapped surfaces do not form in GTR collapse ($2GM/R \leq 1$) is inscribed in the work of Oppenheimer and Snyder too. Unfortunately they overlooked it and as a consequence their final solution (presuming M to be finite) suffers from the anomaly that the space-space component of the metric coefficient e^λ *does not blow up even when the collapse is complete* at $R = 0$ stage (this would blow up iff $M = 0$). In case somebody would like to ignore the above results on unspecified reasons, we attempted to see the consequences of assuming the existence of a finite mass Schwarzschild (rather Hilbert) BH. We considered the problem by considering both (external) Schwarzschild coordinates and all pervading Kruskal coordinates. In either case, we found that, if an EH would exist, the geodesic of a test particle, which must be time like, would become, null there. This means that the EH is no coordinate singularity but the genuine central singularity. Technically, the mass of a SBH, thus must be $M = 0$.

We again found that the acceleration 4-scalar a blows up at the EH indicating that EH is a true central singularity and the mass of SBH must be $M = 0$. When it is so, the curvature scalar $K = 3/4M^4$ does blow up at the EH. Similarly, the components of the Ricci Tensor $\sim M/R^3 \sim 1/M^2$ do blow up at the EH. And the dilemma between the actual Schwarzschild solution and the Hilbert solution for a “point mass” can be resolved only by realizing that gravitational singularities (point masses) must have $M = 0$. We also recall that Rosen [17], in an unambiguous manner noted the impossible and unphysical nature of the $R < 2M$ region:

“ so that in this region R is timelike and T is spacelike. However, this is an impossible situation, for we have seen that R is defined in terms of the circumference of a circle so that R is spacelike, and we are therefore faced with a contradiction. We must conclude that the portion of space corresponding to $R < 2M$ is non-physical. This is a situation which a coordinate transformation even one which removes a singularity can not change. What it means is that the surface $R = 2M$ represents the boundary of physical space and should be regarded as an impenetrable barrier for particles and light rays.”

We have tried to show here that not only the $R < 2M$ region unphysical, it does not exist or is not ever created. And it is well known that Einstein tried to show that BHs are not allowed in GTR [18]. We may recall that the numerical studies of collapse of scalar fields suggest that it is possible to have BHs of $M = 0$ [19]. Also, the supersymmetric string theories find the existence of extremal BHs with charge $Q = M$, which for the chargeless case yields $M = 0$ [20]. Lake pointed out that gravitational singularities could have $M = 0$ [21].

Gravitational collapse of sufficiently massive bodies should indeed result in objects which could be more compact than typical NSs ($z > \sim 0.1$). It is found that, if there are anisotropies, in principle there could be static objects with arbitrary high (but finite) z [21]. Even within the assumption of spherical symmetry, non-standard QCD may allow existence of cold compact objects with masses as large as $10M_{\odot}$ or higher [22]. Such stars are called Q-stars (not the usual quark stars), and they could be much more compact than a canonical NS; for instance, a stable non rotating Q-star of mass $12M_{\odot}$ might have a radius of ~ 52 Km. This may be compared with the value of $R_{gb} \approx 36$ Km of a supposed BH of same mass. And, in any case, when we do away with the assumption of “cold” objects and more importantly, staticity condition there could be objects with arbitrary high z . However, the speed of collapse of such objects, can not be predicted by this work. This work can only tell that, in principle, the collapse process can continue indefinitely because *the ever increasing curvature of spacetime (Ricci Tensor) tends to stretch the physical spacetime to infinite extent.*

In an important work, Chakrabati and Titarchuk [23] suggested that one of the major evidences for the existence of BHCs is that accretion generated X-ray spectrum by them often has a power law tail extending well above 100 KeV. Such a tail may be understood as Compton upscattering of the soft photons around the BHC by the infalling highly relativistic electrons of the accreting plasma [23]. Recall that, although, for NS accretion, the bulk accretion speed can reach upto $\approx 0.5c$, the maximum bulk Lorentz factor works out to be paltry $\gamma \sim 1.1$. Thus the flow is hardly relativistic. This bulk comptonization requires, on the other hand, a modestly relativistic flow with $\gamma < \sim 2$. It turns out that $\gamma = 1 + z$, and an ECO with a value of $z > \sim 1$ could be able to explain the observed hard X-ray tail from the BHCs. On the other hand, for a EH, $\gamma = \infty$, and since a value of $\gamma \sim \text{few}$ is sufficient to explain the observation, this observation may be explained by objects which are not (strict) BHs. There is another line of argument for the having found the existence of EHs in some BHCs [24]. At very low accretion rates, the coupling between electrons and ions could be very weak. In such a case most of the energy of the flow lies with the ions, but since radiative efficiency of ions is very poor, a spherical flow radiates insignificant fraction of accretion energy and carries most of the energy towards the central compact object. Such a flow is called Advection Dominated Flow (ADAF). If the central object has

a “hard surface”, the inflow energy is eventually radiated from the hard surface. On the other hand, if the central object is a BH, the flow energy simply disappears inside the EH. For several supermassive BHCs and stellar mass BHCs, this is claimed to be the case. But there could be several caveats in this interpretation:

- (i) The observed X-ray luminosities for such cases are usually insignificant compared to the corresponding Eddington values (by a factor 10^{-5} to 10^{-7}). Such low luminosities may not be due to accretion at all. Atleast in some cases, they may be due to Synchrotron emission. Recently, Robertson and Leiter [25] have attempted to explain the X-ray emission from several BHCs having even much higher luminosities as Synchrotron origin. Vadawale, Rao & Charrabarti [26] have explained one additional component of hard X-rays from the micro-quasar GRS1915+105 as Synchrotron radiation. The centre of our galaxy is harbours a BHC, Sgr A*, of mass $2.6 \times 10^6 M_\odot$. The recent observation of $\sim 10\text{--}20\%$ linear polarization from this source has strongly suggested against ADAF model [27]. On the other hand, the observed radiation is much more likely due to Synchrotron process [26]. In fact, even more recently, Donato, Ghisellini & Tagliaferri [28] have shown that the low power X-ray emission from the AGNs are due to Synchrotron process rather by accretion process.
- (ii) The x-rays if assumed to be of accretion origin, could be coming from an accretion disk and not from a spherical flow.
- (iii) Even if the X-rays are due to a spherical accretion flow, not in a single case, we have robust independent estimate of the precise accretion rate.
- (iv) Munyaneza & Viollier [29] have claimed that the accurate studies of the motion of stars near Sgr A* are more amenable to a scenario where it is not a BH but a self-gravitating ball of Weakly Interacting Fermions of mass $m_f > \sim 15.9$ keV. Recall here that the Oppenheimer - Volkoff mass limit may be expressed as

$$M_{OV} = 0.54195 M_{pl}^3 m_f^{-2} g_f^{-1/2} = 2.7821 \times 10^9 M_\odot (15 \text{keV}/m_f)^2 (2/g_f)^2 \quad (30)$$

where $M_{Pl} = (\hbar c/G)^{1/2}$ is the PLanck mass and g_f is the degeneracy factor. With a range of $13 < m_f < 17$ keV, these authors point out that the entire range of supermassive BHCs can be understood. Note that the progenitors of the ECOs or BHCs must be much more massive (and larger in size) than those of the NSs. Then it follows from the magnetic flux conservation law that BHCs (at the galactic level) should have magnetic fields considerably higher than NSs. It is also probable even when they are old, their diminished magnetic fields are considerably higher than 10^{10} G. In such cases, BHCs will not exhibit Type I X-ray burst activity. There may indeed be evidence for intrinsic (high) magnetic fields for the BHCs [25]. However, in some cases, they may well have sufficiently low magnetic field and show Type I bursts. It is now known that Cir X-1 which was considered a BHC, did show Type I burst, a signature of “hard surface”. Irrespective of interprtation of presently available observations, our work has shown that the BHCs can not be, in a strict sense, (finite mass) BHs because then *timelike geodesics would become null* on their EHs. If a BH would exist, the proper length of an infalling astronomer or anything would become ∞ as it would approach the central singularity. Then how would the observer stay put in a geometrical point? Such inconsistency actually does not arise because we have shown that one has to traverse infinite proper length over infinite proper time in order to reach the central singularity ($M = 0$).

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